

A NEW LAPLACIAN COMPARISON THEOREM AND ITS APPLICATIONS ON FINSLER MANIFOLD

MEI GUAN AND SONGTING YIN*

Abstract. For a Finsler manifold equipped with the weighted volume form, we established a new Laplacian comparison theorem for the distance function. As applications, we obtain Bishop-Gromov type volume comparison for the geodesic balls, and Cheng type comparison theorem for the first Dirichlet eigenvalue. Moreover, we also give a simple proof for Myers theorem.

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