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SOME IMPROVEMENTS ABOUT NUMERICAL RADIUS INEQUALITIES FOR HILBERT SPACE OPERATORS

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Abstract. In this paper, we give several numerical radius inequalities for Hilbert space operators by using the Young-type inequalities and the generalization of the Buzano of the Schwarz inequality. These inequalities improve the classical numerical radius inequalities. We also give the upper bound of the numerical radius of 2×2 operator matrices.

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