

GENERALIZATIONS OF HARDY–TYPE INEQUALITIES BY THE HERMITE INTERPOLATING POLYNOMIAL

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Abstract. In this paper we obtain generalizations of Hardy-type inequalities for convex functions of the higher order by applying Hermite interpolating polynomials. The results for particular cases: Lagrange, $(m, n - m)$ and two-point Taylor interpolating polynomials are also considered. Finally, we derive the Grüss and Ostrowski type inequalities related to these generalizations.

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REFERENCES

- [1] M. ADIL KHAN, S. IVELIĆ BRADANOVIĆ AND J. PEČARIĆ, *Generalizations of Sherman's inequality by Hermite's interpolating polynomial*, Math. Inequal. Appl., **19**, no. 4 (2016), 1181–1192.
- [2] R. P. AGARWAL, S. IVELIĆ BRADANOVIĆ AND J. PEČARIĆ, *Generalizations of Sherman's inequality by Lidstone's interpolation polynomial*, J. Inequal. Appl. **6**, 2016 (2016).
- [3] R. P. AGARWAL, P. J. Y. WONG, *Error Inequalities in Polynomial Interpolation and Their Applications*, Kluwer Academic Publishers, Dordrecht/Boston/London, 1993.
- [4] K. E. ATKINSON, *An Introduction to Numerical Analysis*, 2nd ed., Wiley, New York, 1989.
- [5] P. CERONE, S. S. DRAGOMIR, *Some new Ostrowski-type bounds for the Chebyshev functional and applications*, J. Math. Inequal. **8** (1) (2014), 159–170.
- [6] P. L. CHEBYSHEV, *Sur les expressions approximatives des integrales definies par les autres prises entre les memes limites*, Proc. Math. Soc. Charkov, **2**, (1882) 93–98.
- [7] G. H. HARDY, *Notes on some points in the integral calculus LX: An inequality between integrals (60)*, Messenger of Math. **54** (1925), 150–156.
- [8] S. IVELIĆ BRADANOVIĆ, NAVEED LATIF, J. PEČARIĆ, *Generalizations of Sherman's inequality by Taylor's formula*, Journal of Inequalities and Special functions, vol. 8, issue 2 (2017), 18–30.
- [9] S. IVELIĆ BRADANOVIĆ, J. PEČARIĆ, *Generalizations of Sherman's inequality*, Per. Math. Hung. **74**, 197–219 (2017).
- [10] K. KRULIĆ HIMMELREICH, *Generalizations of Hardy Type Inequalities by Taylor's Formula*, Mathematica Slovaca **72** (1) (2022), 67–84.
- [11] K. KRULIĆ, J. PEČARIĆ, AND L.-E. PERSSON, *Some new Hardy–type inequalities with general kernels*, Math. Inequal. Appl. **12** (3) (2009), 473–485.
- [12] K. KRULIĆ, J. PEČARIĆ, *Some new Hardy–type inequalities with general kernels II*, Math. Inequal. Appl. **19** (1) (2016), 73–84.
- [13] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, *Inequalities of Hardy and Jensen*, Element, Zagreb, 2013.
- [14] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, M. PRALJAK, *Generalizations of Hardy Type Inequalities by Abel-Gontscharoff's Interpolating Polynomial*, Mathematics 2021, **9**, 1724.
- [15] K. KRULIĆ HIMMELREICH, J. PEČARIĆ, D. POKAZ, M. PRALJAK, *Hardy-type inequalities generalized via Montgomery identity*, accepted for publication, Montes Taurus J. Pure Appl. Math. **6** (3), (2024), 62–71.

- [16] A. KUFNER, L. MALIGRANDA, AND L.-E. PERSSON, *The prehistory of the Hardy inequality*, Amer. Math. Monthly **113** (2006), no. 8, 715–732.
- [17] A. KUFNER, L. MALIGRANDA, AND L.-E. PERSSON, *The Hardy Inequality. About its History and Some Related Results*, Vydavatelský Servis Publishing House, Pilsen, 2007.
- [18] J. E. PEČARIĆ, F. PROSCHAN, AND Y. L. TONG, *Convex functions, Partial Orderings and Statistical Applications*, Academic Press, San Diego, 1992.
- [19] D. POKAZ, *Inequality of Hardy-type for n -convex function via interpolation polynomial and Green functions*, Math. Inequal. Appl. **26** (4) (2023), 965–976.
- [20] T. POPOVICIU, *Introduction à la théorie des différences divisées*, Bull. Mathem., Societatea Romana de Stiinte, Bukharest 42, 1940, 65–78.