

SOME WEIGHTED DYNAMIC INEQUALITIES OF HARDY TYPE WITH KERNELS ON TIME SCALES NABLA CALCULUS

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Abstract. In this paper, we present some properties of the time scale nabla calculus and how to apply it for proving the dynamic inequalities. Also, we prove some weighted dynamic inequalities of Hardy type with kernels on time scales nabla calculus and also, we study the characterizations of the weights for these inequalities in different spaces and for the exponent $p > 1$. The Holder inequality, Jensen inequality, and Minkowski inequality are used to prove our results.

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