

WIGNER—YANASE—DYSON FUNCTION AND LOGARITHMIC MEAN

SHIGERU FURUICHI

Abstract. The ordering between Wigner–Yanase–Dyson function and logarithmic mean is known. Also bounds for logarithmic mean are known. In this paper, we give two reverse inequalities for Wigner–Yanase–Dyson function and logarithmic mean. We also compare the obtained results with the known bounds of the logarithmic mean. Finally, we give operator inequalities based on the obtained results.

Mathematics subject classification (2020): Primary 26E60; Secondary 26D07.

Keywords and phrases: Wigner–Yanase–Dyson function, logarithmic mean, Kantorovich constant, Specht ratio and reverse inequalities.

REFERENCES

- [1] S. FURUICHI, *Unitarily invariant norm inequalities for some means*, J. Inequal. Appl., **2014** (2014), Art. 158.
- [2] S. FURUICHI AND M. E. AMLASHI, *On bounds of logarithmic mean and mean inequality chain*, arXiv:2203.01134.
- [3] S. FURUICHI AND H. R. MORADI, *Advances in mathematical inequalities*, De Gruyter, 2020.
- [4] S. FURUICHI AND K. YANAGI, *Schrödinger uncertainty relation, Wigner–Yanase–Dyson skew information and metric adjusted correlation measure*, J. Math. Anal. Appl., **388** (2) (2012), 1147–1156.
- [5] P. GIBILISCO, F. HANSEN AND T. ISOLA, *On a correspondence between regular and non-regular operator monotone functions*, Linear Algebra Appl., **430** (2009), 2225–2232.
- [6] F. HANSEN, *Metric adjusted skew information*, Proc. Nat. Acad. Sci., **105** (2008), 9909–9916.
- [7] F. HIAI AND H. KOSAKI, *Means for matrices and comparison of their norms*, Indiana Univ. Math. J. **48** (1999), 899–936.
- [8] F. HIAI AND H. KOSAKI, *Means of Hilbert space operators*, Springer–Verlag, 2003.
- [9] F. HIAI, H. KOSAKI, D. PETZ AND B. RUSKAI, *Families of completely positive maps associated with monotone metrics*, Linear Algebra Appl., **48** (439) (2013), 1749–1791.
- [10] L. V. KANTOROVICH, *Functional analysis and applied mathematics*, Uspekhi Mat. Nauk, **3**: 6 (28) (1948), 89–185, <http://mi.mathnet.ru/eng/umn/v3/i6/p89>.
- [11] H. KOSAKI, *Positive definiteness and infinite divisibility of certain functions of hyperbolic cosine function*, Internat. J. Math., **33** (7) (2022), 2250050.
- [12] H. KOSAKI, *Positive definiteness of functions with applications to operator norm inequalities*, Mem. Amer. Math. Soc., **212** (997), 2011.
- [13] H. KOSAKI, *Strong monotonicity for various means*, J. Func. Anal., **267** (2014), 1917–1958.
- [14] D. PETZ AND H. HASEGAWA, *On the Riemannian metric of α -entropies of density matrices*, Lett. Math. Phys., **38** (1996), 221–225.
- [15] W. SPECHT, *Zur Theorie der elementaren Mittel*, Math. Z., **74** (1960), 91–98, [10.1007/BF01180475](https://doi.org/10.1007/BF01180475).
- [16] V. E. S. SZABÓ, *A class of matrix monotone functions*, Linear Algebra Appl., **420** (2007), 79–85.