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SPLITTING INEQUALITIES FOR DIFFERENCES OF EXPONENTIALS

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Abstract. The paper is focused on two-sided splitting inequalities for differences of complex exponentials

$$\left|\Delta^k e^{itf(n)}\right|, \ k \in \mathbb{N}, \ t \in \mathbb{R},$$

for large $n \in \mathbb{N}$, where $\{f(n)\}_{n=1}^{\infty}$ is real unbounded sequence clustering with appropriate speed. Moreover, it is shown that if $\{e_n\}_{n\in\mathbb{N}}$ is a Riesz basis of a Hilbert space H, then for any $k \geqslant 1$ the system $\left\{\Delta^k e_n\right\}_{n\in\mathbb{N}}$ is complete, minimal but not uniformly minimal in H. Also some properties of systems of functions of real argument t,

$$\left\{\Delta^k e^{itf(n)}\right\}_{n\in\mathbb{N}},$$

where $k \in \mathbb{N} \cup \{0\}$, are discussed.

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