IMPROVEMENTS OF A–NUMERICAL RADIUS FOR SEMI–HILBERTIAN SPACE OPERATORS

Hongwei Qiao, Guojun Hai* and Alatancang Chen

Abstract. Let $A$ be a bounded positive operator on a complex Hilbert space $(H, \langle \cdot, \cdot \rangle)$. The semi-product $\langle x, y \rangle_A := \langle Ax, y \rangle, x, y \in H$, induces a semi-norm $\| \cdot \|_A$ on $H$. Let $\omega_A(T)$ and $\|T\|_A$ denote the A-numerical radius and the A-operator semi-norm of an operator $T$ in semi-Hilbertian space $(H, \langle \cdot, \cdot \rangle_A)$, respectively. In this paper, some new bounds for the A-numerical radius of operators in semi-inner product space induced by $A$ are derived. In particular, for $T \in B_A(H)$ and $\alpha \geq 0$, we prove that

$$\omega_A^4(T) \leq \frac{1+2\alpha}{16(1+\alpha)} \|T^\sharp_A T + TT^\sharp_A\|_A^2 + \frac{3+8\alpha}{8(1+\alpha)} \|T^\sharp_A + TT^\sharp_A\|_A \omega_A(T^2)$$

and

$$\omega_A^2(T) \leq \frac{1+2\alpha}{8(1+\alpha)} \|T^\sharp_A T + TT^\sharp_A\|_A^2 + \frac{1}{2(1+\alpha)} \omega_A^2(T^2).$$

It is worth noting that our results improve the existing A-numerical radius inequalities. Further, we also give a refinement inequality of A-operator semi-norm triangle inequality.

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REFERENCES


