

RADII OF LEMNISCATE STARLIKENESS AND CONVEXITY OF THE FUNCTIONS INCLUDING DERIVATIVES OF BESSEL FUNCTIONS

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Abstract. In this paper, our aim is to determine the radii of starlikeness and convexity associated with lemniscate of Bernoulli for three different kinds of normalizations of the function $N_V(z) = az^2 J_V''(z) + bz J_V'(z) + c J_V(z)$, where J_V is the Bessel function of the first kind of order v . The key tools in the proof of our main results are the Mittag-Leffler expansion for the function $N_V(z)$ and properties of real zeros of it. Also, we give tables related with special cases of parameters.

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REFERENCES

- [1] Á. BARICZ, P. A. KUPÁN, R. SZÀSZ, *The radius of starlikeness of normalized Bessel functions of the first kind*, Proc. Amer. Math. Soc., **142**, 5 (2014), 2019–2025.
- [2] Á. BARICZ, R. SZÀSZ, *The radius of convexity of normalized Bessel functions of the first kind*, Anal. Appl., **12**, 5 (2014), 485–509.
- [3] Á. BARICZ, R. SZÀSZ, *The radius of convexity of normalized Bessel functions*, Anal. Math., **41**, 3 (2015), 141–151.
- [4] Á. BARICZ, M. ÇAĞLAR, E. DENIZ, *Starlikeness of Bessel functions and their derivatives*, Math. Inequal. Appl., **19**, 2 (2016), 439–449.
- [5] Á. BARICZ, H. ORHAN, R. SZÀSZ, *The radius of α -convexity of normalized Bessel functions of the first kind*, Comput. Method. Func. Theo., **16**, 1 (2016), 93–103.
- [6] R. K. BROWN, *Univalence of Bessel functions*, Proc. Amer. Math. Soc., **11**, 2 (1960), 278–283.
- [7] M. ÇAĞLAR, E. DENIZ, R. SZÀSZ, *Radii of α -convexity of some normalized Bessel functions of the first kind*, Results Math., **72**, (2017), 2023–2035.
- [8] E. DENIZ, R. SZÀSZ, *The radius of uniform convexity of Bessel functions*, J. Math. Anal. Appl., **453**, 1 (2017), 572–588.
- [9] E. DENIZ, S. KAZIMOĞLU, M. ÇAĞLAR, *Radii of starlikeness and convexity of Bessel function derivatives*, Ukrainian Math. J., **73**, 11 (2022), 1686–1711.
- [10] M. E. H. ISMAIL, M. E. MULDOON, *Bounds for the small real and purely imaginary zeros of Bessel and related functions*, Meth. Appl. Anal., **2**, 1 (1995), 1–21.
- [11] S. KAZIMOĞLU, E. DENIZ, *The radii of starlikeness and convexity of the functions including derivatives of Bessel functions*, Turk. J. Math., **46**, (2022), 894–911.
- [12] S. KAZIMOĞLU, E. DENIZ, *Radius Problems for functions containing derivatives of Bessel functions*, Comput. Method. Func. Theo., **23**, (2023), 421–446.
- [13] S. KAZIMOĞLU, E. DENIZ, *Radii problems for the function $az^2 J_V''(z) + bz J_V'(z) + c J_V(z)$* , Math. Inequal. Appl., **26**, 1 (2023), 47–67.
- [14] E. KREYSZIG, J. TODD, *The radius of univalence of Bessel functions*, Illinois J. Math., **4**, (1960), 143–149.
- [15] W. C. MA, D. MINDA, *A unified treatment of some special classes of univalent functions*, In: Proceedings of the Conference on Complex Analysis (Tianjin, 1992), Conf. Proc. Lecture Notes Anal. I, Int. Press, Cambridge, MA, pp. 157–169.

- [16] V. MADAAN, A. KUMAR, V. RAVICHANDRAN, *Radii of starlikeness and convexity of some entire functions*, Bull. Malays. Math. Sci. Soc., **43**, (2020), 4335–4359.
- [17] A. MCD. MERCER, *The zeros of $az^2J'_\nu(z) + bzJ'_\nu(z) + cJ_\nu(z)$ as functions of order*, Internat. J. Math. Math. Sci., **15**, (1992), 319–322.
- [18] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT, C. W. CLARK (Eds.), *NIST Handbook of Mathematical Functions*, Cambridge Univ. Press, Cambridge, 2010.
- [19] S. M. SHAH, S. Y. TRIMBLE, *Entire functions with univalent derivatives*, J. Math. Anal. Appl., **33**, (1971), 220–229.
- [20] J. SOKÓL, J. STANKIEWICZ, *Radius of convexity of some subclasses of strongly starlike functions*, Zeszyty Nauk. Politech., Rzeszowskiej Mat., **19**, (1996), 101–105.
- [21] R. SZÀSZ, *About the radius of starlikeness of Bessel functions of the first kind*, Monatsh. Math., **176**, (2015), 323–330.