

## IMPROVED JENSEN–DRAGOMIR TYPE INEQUALITIES AND APPLICATIONS

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*Abstract.* In this paper we establish refinements of the Jensen–Dragomir type inequalities for convex and log-convex functions. Some further generalizations of these types of inequalities via the theory of weak submajorization are also given. Several applications of the obtained inequalities for refining and reversing of the majorization inequality and the generalized triangle inequality in Banach spaces are also presented.

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