

IMPROVED INEQUALITIES FOR THE BEREZIN NUMBER

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Abstract. A functional Hilbert space is the Hilbert space of complex-valued functions on some set $\Theta \subseteq \mathbb{C}$ that the evaluation functionals $\varphi_\tau(f) = f(\tau)$, $\tau \in \Theta$ are continuous on \mathcal{H} . The Berezin number(radius) of an operator T is defined by $\mathbf{ber}(T) = \sup_{\tau \in \Theta} |T\hat{k}_\tau, \hat{k}_\tau|$, where the operator T acts on the reproducing kernel Hilbert space $\mathcal{H} = \mathcal{H}(\Theta)$ over some(non-empty) set Θ . In this paper, we give some Berezin number inequalities. Moreover, we present some inequalities involving the weighted Berezin number of operators on the reproducing kernel Hilbert space.

Mathematics subject classification (2020): Primary 47A63; Secondary 15A60, 47A60.

Keywords and phrases: Berezin radius, Berezin norm, reproducing kernel, Schwarz inequality.

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