

ON COMPACTNESS OF OPERATORS FROM BANACH  
SPACES OF HOLOMORPHIC FUNCTIONS TO BANACH SPACES

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*Abstract.* We investigate a widely used application of compactness of bounded linear operators  $T: X(\mathbb{B}) \rightarrow Y$ , where  $X(\mathbb{B})$  is a Banach space of holomorphic functions on the open unit ball  $\mathbb{B} \subset \mathbb{C}^N$  and  $Y$  is a Banach space. In particular, we show that compactness of the operator when  $X(\mathbb{B})$  is not reflexive, is not a sufficient condition for the property that every bounded sequence  $(f_n)_{n \in \mathbb{N}}$  in  $X(\mathbb{B})$  such that  $f_n \rightarrow 0$  with respect to the compact open topology as  $n \rightarrow \infty$ , implies that  $T(f_n) \rightarrow 0$  with respect to the norm of  $Y$  as  $n \rightarrow \infty$ .

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