

## HIGHER ORDER POINCARÉ INEQUALITY AND CACCIOPPOLI INEQUALITY WITH ORLICZ NORMS FOR DIFFERENTIAL FORMS

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**Abstract.** In this paper, we establish the local higher order Poincaré inequality and Caccioppoli inequality with Orlicz norms for solutions to the non-homogeneous  $A$ -harmonic equations on differential forms. Moreover, the global higher order Poincaré inequality and Caccioppoli inequality are derived. As applications, the higher order Caccioppoli-type inequality and a weak type inequality for homotopy operator are obtained.

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### REFERENCES

- [1] R. P. AGARWAL, S. DING AND C. A. NOLDER, *Inequalities for differential forms*, Springer, 2009.
- [2] J. M. BALL, *Convexity conditions and existence theorems in nonlinear elasticity*, Arch. Rational Mech. Anal. **63** (1977), 337–403.
- [3] J. M. BALL AND F. MURAT,  $W^{1,p}$  quasi-convexity and variational problems for multiple integrals, J. Funct. Anal. **58** (1984), 225–253.
- [4] S. M. BUCKLEY, J. J. MANFREDI AND E. VILLAMOR, *Regularity theory and traces of  $A$ -harmonic functions*, Trans. Amer. Math. Soc. **348** (1996), 1–12.
- [5] W. S. COHN, G. LU AND S. LU, *Higher order Poincaré inequalities associated with linear operators on stratified groups and applications*, Mathematische Zeitschrift **244** (2003), 309–335.
- [6] S. DING, *Two-weight Caccioppoli inequalities for solutions of nonhomogeneous  $A$ -harmonic equations on Riemannian manifolds*, Proc. Amer. Math. Soc. **132** (2004), 2367–2375.
- [7] S. DING AND B. LIU, *Dirac-harmonic equations for differential forms*, Nonlinear Anal. Theor. **122** (2015), 43–57.
- [8] S. DING AND C. A. NOLDER, *Weighted Poincaré inequalities for solutions to  $A$ -harmonic equations*, Illinois J. Math. **46** (2002), 199–205.
- [9] S. DING, G. SHI AND Y. XING, *Higher integrability of iterated operators on differential forms*, Nonlinear Anal. Theor. **145** (2016), 83–96.
- [10] R. FINN AND J. SERRIN, *On the Hölder continuity of quasi-conformal and elliptic mappings*, Trans. Amer. Math. Soc. **89** (1958), 1–15.
- [11] N. FUSCO AND C. SBORDONE, *Higher integrability of the gradient of minimizers of functionals with nonstandard growth conditions*, Commun. Pure Appl. Math. **43** (1990), 673–683.
- [12] F. W. GEHRING, *The  $L^p$ -integrability of partial derivatives of a quasiconforming mappings*, Acta Mathematica **130** (1973), 265–277.
- [13] T. IWANIEC AND A. LUTOBORSKI, *Integral estimates for null Lagrangians*, Arch. Ration. Mech. Anal. **125** (1993), 25–79.
- [14] P. LI AND J. WANG, *Hölder estimates and regularity for holomorphic and harmonic functions*, J. Differ. Geom. **58** (2001), 309–329.
- [15] B. LIU,  $A_r(\Omega)$ -weighted imbedding inequalities for  $A$ -harmonic tensors, J. Math. Anal. Appl. **273** (2002), 667–676.
- [16] G. LU, *Polynomials, Higher order sobolev extension theorems and interpolation inequalities on weighted Folland-Stein spaces on stratified Groups*, Acta Mathematica Sinica. **16** (2000), 405–444.

- [17] Y. LV, *Poincaré inequalities and the sharp maximal inequalities with  $-$ -norms for differential forms*, J. Inequal. Appl. **1** (2013), 1–11.
- [18] G. LU AND R. L. WHEEDEN, *High order representation formulas and embedding theorems on stratified groups and generalizations*, Studia Mathematica **142** (2000), 101–133.
- [19] J. NIU AND Y. XING, *The higher integrability of commutators of Calderón-Zygmund singular integral operators on differential forms*, J. Funct. Spaces. **2018** (2018), 1–9.
- [20] C. A. NOLDER, *Hardy-Littlewood theorems for  $A$ -harmonic tensors*, Illinois J. Math. **43** (1999), 613–632.
- [21] E. W. STRDULINSKY, *Higher integrability from reverse Hölder inequalities*, Indiana University Mathematics Journal **28** (1980), 407–413.
- [22] Y. WANG AND C. WU, *Sobolev imbedding theorems and Poincaré inequalities for Green's operator on solutions of the nonhomogeneous  $A$ -harmonic equation*, Comput. Math. Appl. **47** (2004), 1545–1554.
- [23] Y. WANG, *Two-weight Poincaré type inequalities for differential forms in  $L^s(\mu)$ -averaging domains*, Appl. Math. Lett. **20** (2007), 1161–1166.
- [24] Y. XING, *Weighted Poincaré-type estimates for conjugate  $A$ -harmonic tensors*, J. Inequal. Appl. **1** (2005), 1–6.
- [25] Y. XING, *Weighted integral inequalities for solutions of the  $A$ -harmonic equation*, J. Math. Anal. Appl. **279** (2003), 350–363.
- [26] Y. XING AND S. DING, *Higher integrability of Green's operator and Homotopy operator*, J. Math. Anal. Appl. **446** (2017), 648–662.
- [27] Y. XING AND S. DING, *Caccioppoli inequalities with Orlicz norms for solutions of harmonic equations and applications*, Nonlinearity, **23** (2010) 1109–1119.