

ESTIMATES FOR p -ADIC HARDY OPERATORS ON VARIABLE EXPONENT MORREY–HERZ TYPE SPACES

KIEU HUU DUNG*, TRAN LUU CUONG AND PHAM THI KIM THUY

Abstract. In this paper, we establish some necessary and sufficient conditions for the boundedness of p -adic Hardy–Cesàro operators on central Morrey, Herz, and Morrey–Herz spaces with variable exponents. Finally, the boundedness of rough p -adic Hardy operators on variable exponent Morrey–Herz spaces is also discussed.

Mathematics subject classification (2020): 42B35, 47G10, 26D15.

Keywords and phrases: Rough p -adic Hardy operator, p -adic Hardy–Cesàro operator, Morrey–Herz space, variable exponent, p -adic analysis.

REFERENCES

- [1] A. ALMEIDA AND D. DRIHEM, *Maximal, Potential and singular type operators on Herz spaces with variable exponents*, Journal of Mathematical Analysis and Applications, **394**, 781–795 (2012).
- [2] S. ALBEVERIO, A. YU. KHRENNIKOV AND V. M. SHEPKOVICH, *Harmonic analysis in the p -adic Lizorkin spaces: fractional operators, pseudo-differential equations, p -wavelets, Tauberian theorems*, Journal of Fourier Analysis and Applications, **12** (4), 393–425 (2006).
- [3] R. A. BANDALIEV, *The boundedness of multidimensional Hardy operators in weighted variable Lebesgue spaces*, Lithuanian Mathematical Journal, **50**, 249–259 (2010).
- [4] D. CRUZ-URIBE AND A. FIORENZA, *Variable Lebesgue Spaces: Foundations and Harmonic Analysis*, Springer-BaseL, 2013.
- [5] L. F. CHACÓN-CORTÉS AND H. RAFEIRO, *Variable exponent Lebesgue spaces and Hardy–Littlewood maximal function on p -adic numbers*, p -Adic Numbers, Ultrametric Analysis and Applications, **12**, 90–111 (2020).
- [6] M. CHRIST AND L. GRAFAKOS, *Best constants for two non-convolution inequalities*, Proceedings of the American Mathematical Society, **123**, 1687–1693 (1995).
- [7] N. M. CHUONG, D. V. DUONG, *Weighted Hardy–Littlewood operators and commutators on p -adic functional spaces*, Adic Numbers, Ultrametric Analysis and Applications, **5**, 65–82 (2013).
- [8] N. M. CHUONG, D. V. DUONG AND K. H. DUNG, *Multilinear Hausdorff operator on variable exponent Morrey–Herz type spaces*, Integral Transforms and Special Functions, **31** (1), 62–86 (2020).
- [9] N. M. CHUONG AND H. D. HUNG, *Bounds of weighted Hardy–Cesàro operators on weighted Lebesgue and BMO spaces*, Integral Transforms and Special Functions, **25**, 697–710 (2014).
- [10] K. H. DUNG, D. L. C. MINH, T. T. NANG, *Boundedness of Hardy–Cesàro operators on variable exponent Morrey–Herz spaces*, Filomat, **37** (4), 1001–1016 (2023).
- [11] L. DIENING, M. RUŽIČKA, *Calderón–Zygmund operators on generalized Lebesgue spaces $L^{p(x)}$ and problems related to fluid dynamics*, Journal für die Reine und Angewandte Mathematik, **563**, 197–220 (2003).
- [12] L. DIENING, S. SAMKO, *Hardy-inequality in variable exponent Lebesgue spaces*, Fractional Calculus and Applied Analysis, **10** (1), 1–18 (2007).
- [13] D. E. EDMUNDS AND A. MESKHI, *Two-weighted Hardy operators in $L^p(\cdot)$ spaces and applications*, Studia Mathematica, **249**, 143–162 (2019).
- [14] Z. W. FU, S. Z. LU AND F. Y. ZHAO, *Commutators of n -dimensional rough Hardy operator*, Science China Mathematics, **54** (1), 95–104 (2011).

- [15] Z. W. FU, S. Z. LU AND S. G. SHI, *Two characterizations of central BMO space via the commutators of Hardy operators*, Forum Mathematicum, **33** (2), 505–529 (2021).
- [16] G. GAO, *Boundedness for commutators of n -dimensional rough Hardy operators on Morrey-Herz spaces*, Comput. Math. Appl. **64** (4), 544–549 (2012).
- [17] G. GAO AND Y. ZHONG, *Some estimates of Hardy operators and their commutators on Morrey-Herz spaces*, Journal of Mathematical Inequalities, **11** (1), 49–58 (2017).
- [18] A. HUSSAIN, N. SARFRAZ, I. KHAN, A. ALSUBIE AND N. N. HAMADNEH, *The boundedness of commutators of rough p -adic fractional Hardy type operators on Herz type spaces*, Journal of Inequalities and Applications, **123**, (2021).
- [19] G. H. HARDY, *Note on a theorem of Hilbert*, Mathematische Zeitschrift, **6**, 314–317 (1920).
- [20] H. D. HUNG, *The p -adic weighted Hardy-Cesàro operator and an application to discrete Hardy inequalities*, Journal of Mathematical Analysis and Applications, **409**, 868–879 (2014).
- [21] H. D. HUNG AND L. D. KY, *New weighted multilinear operators and commutators of Hardy-Cesàro type*, Acta Math. Sci. Ser. B (Engl. Ed.), **35** (6), 1411–1425 (2015).
- [22] M. IZUKI, *Fractional integrals on Herz-Morrey spaces with variable exponent*, Hiroshima Mathematical Journal, **40** (3), 343–355 (2010).
- [23] M. IZUKI, E. NAKAI, Y. SAWANO, *Function spaces with variable exponents. An Introduction*, Scientiae Mathematicae Japonicae, **77** (2), 187–315 (2014).
- [24] A. YU. KHRENNIKOV, *p -adic valued distributions in mathematical physics*, Kluwer Academic Publishers, Dordrecht-Boston-London, 1994.
- [25] A. KOCHUBEI, *Radial solutions of non-Archimedean pseudo-differential equations*, Pacific Journal of Mathematics, **269**, 355–369 (2014).
- [26] S. V. KOZYREV, *Methods and applications of ultrametric and p -adic analysis: From wavelet theory to biophysics*, Proceedings of the Steklov Institute of Mathematics, **274**, 1–84 (2011).
- [27] W. LI, T. ZHANG AND L. XUE, *Two-weight inequalities for Hardy operator and commutators*, Journal of Mathematical Inequalities, **9** (3), 653–664 (2015).
- [28] J. A. OGUNTUASE, C. A. OKPOTI, L. E. PERSSON AND F. K. A. ALLOTEY, *Multidimensional Hardy type inequalities for $p < 0$ and $0 < p < 1$* , Journal of Mathematical Inequalities, **1** (1), 1–11 (2007).
- [29] V. D. PROKHOROV, *Lorentz norm inequalities for the Hardy operator involving suprema*, Proceedings of the American Mathematical Society, **140** (5), 1585–1592 (2012).
- [30] K. S. RIM AND J. LEE, *Estimates of weighted Hardy-Littlewood averages on the p -adic vector space*, Journal of Mathematical Analysis and Applications, **324** (2), 1470–1477 (2006).
- [31] E. SAWYER, *Weighted Lebesgue and Lorentz norm inequalities for the Hardy operator*, Transactions of the American Mathematical Society, **281** (1), 329–337 (1984).
- [32] V. D. STEPANOV, *The weighted Hardy's inequality for nonincreasing functions*, Transactions of the American Mathematical Society, **338** (1), 173–186 (1993).
- [33] V. S. VLADIMIROV, I. V. VOLOVICH AND E. I. ZELENOV, *p -adic analysis and mathematical physics*, World Scientific, 1994.
- [34] S. S. VOLOSIVETS, *Hausdorff operators on p -adic linear spaces and their properties in Hardy, BMO, and Hölder spaces*, Mathematical Notes, **3**, 382–391 (2013).
- [35] J. L. WU AND W. J. ZHAO, *Boundedness for fractional Hardy-type operator on variable-exponent Herz-Morrey spaces*, Kyoto Journal of Mathematics, **56**, 831–845 (2016).
- [36] Q. Y. WU, L. MI, Z. W. FU, *Boundedness of p -adic Hardy operators and their commutators on p -adic central Morrey and BMO spaces*, Journal of Function Spaces, **2013**, Article ID 359193, 10 pages, (2013).