

## INEQUALITIES FOR THE DERIVATIVE AND POLAR DERIVATIVE OF LACUNARY POLYNOMIALS

NUTTAPONG ARUNRAT AND KEAITSUDA MANEERUK NAKPRASIT\*

*Abstract.* In this paper, we study lacunary polynomials of degree  $n$  having  $s$ -fold zeros at the origin and the remaining zeros lying on or outside the boundary of a prescribed disk. This study in turns gives generalizations and improvements of some well-known results. Besides, we also generalize as well as improves upon a result due to Aziz and Shah by extending it to the polar derivative.

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