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SPECTRAL ANALYSIS AND INEQUALITY BOUNDS FOR HEIGHT AND DETERMINANT FUNCTIONS

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Abstract. Spectral height and determinant are not invariant under operator or metric deformations. The variations of spectral height and determinant of Laplacian under conformal change of metric in two dimensional Riemannian manifolds are computed explicitly using Polyakov formula. However, in dimensions higher than two, there are no such formula for computing the conformal or other such variations. In this work, we extend the Polyakov formula to study some generic and conformal variations of the height and determinant functions on closed Riemannian manifolds in higher dimensions and found their spectral inequality bounds.

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REFERENCES

[1] W. ARENDT, *A characterisation of generators of positive semigroups*, Proceedings of the Royal Irish Academy, Section A: Mathematical and Physical Sciences **84A**, 1 (1984), 155–174.

[2] E. W. BARNES, *The genesis of the double gamma functions*, Proceedings of the London Mathematical Society **1**, 1 (1899), 358–81.

[3] JUNESANG CHOI, *Determinants of the Laplacians on the n-dimensional unit sphere S^n*, Advances in Difference Equations **2013**, 1 (2013), 1–12.

[4] JUNESANG CHOI AND H. M. SRIVASTAVA, *An application of the theory of the double Gamma function*, Kyushu Journal of Mathematics **53**, 1 (1999), 209–222.

[5] LAHOUCINE ELAISSAOUI, *On expansions involving the Riemann zeta function and its derivatives*, Journal of Mathematical Analysis and Applications **517**, 1 (2023), 126569.

[6] PETER B. GILKEY, *Invariance theory: the heat equation and the Atiyah-Singer index theorem*, vol. 16, CRC press, 2018.

[7] ALEXANDER GRIGORYAN, *Heat kernel and analysis on manifolds*, vol. 47, American Mathematical Soc., 2009.

[8] SORA HIRAMATSU AND YUKI SEO, *Determinant for positive operators and Oppenheim's inequality Journal of Mathematical Inequalities*, **15**, 4 (2021), 1637–1645.

[9] K. KIRSTEN AND ALAN MCKANE, *Functional determinants in the presence of zero modes. Quantum Field Theory under the Influence of External Conditions*, Rinton Press Incorporated, 2004.

[10] H. KUMAGAI, *The determinant of the Laplacian on the n-sphere*, Acta Arithmetica **91**, 3 (1999), 199–208.

[11] JOHN M. LEE, *Smooth manifolds*, Springer, New York, 2012.

[12] SUBBARAMIAH MINAKSHISUNDARAM AND ÅKE PLEIJEL, *Some properties of the eigenfunctions of the Laplace-operator on Riemannian manifolds*, Canadian Journal of Mathematics **1**, 3 (1949), 242–256.

[13] YOICHI MOTOHASHI, *Spectral theory of the Riemann zeta-function*, no. 127, Cambridge University Press, 1997.

[14] WERNER MÜLLER, *Relative zeta functions, relative determinants and scattering theory*, Communications in mathematical physics **192**, 1 (1998), 309–347.

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[15] KATE OKIKIOLU, *Critical metrics for the determinant of the Laplacian in odd dimensions*, Annals of Mathematics **153**, 1 (2001), 471–531.

[16] LOUIS OMENYI, *Meromorphic continuation of the spectral zeta kernel*, General Mathematics Notes **36**, 2 (2016), 26–37.

[17] L. OMENYI AND M. UCHENNA, *Global analysis on Riemannian manifolds*, The Australian Journal of Mathematical Analysis and Applications **16**, 2 (2019), 1–17.

[18] LOUIS OMENYI AND MCSYLVESTER OMABA, *Some domination inequalities for spectral zeta kernels on closed Riemannian manifolds*, Journal of Mathematical Inequalities **16**, 4 (2022), 1525–1539.

[19] BRAD OSGOOD, RALPH PHILLIPS AND PETER SARNAK, *Extremals of determinants of Laplacians*, Journal of functional analysis **80**, 1 (1988), 148–211.

[20] DANIEL B. RAY AND ISADORE M. SINGER, *R-torsion and the Laplacian on Riemannian manifolds*, Advances in Mathematics **7**, 2 (1971), 145–210.

[21] KEN RICHARDSON, *Critical points of the determinant of the Laplace operator*, Journal of Functional Analysis **122**, 1 (1994), 52–83.

[22] STEVEN ROSENBERG, *The Laplacian on a Riemannian manifold: an introduction to analysis on manifolds*, no. 31, Cambridge University Press, 1997.

[23] H. M. SRIVASTAVA, KHALED MEHREZ AND ZIVORAD TOMOVSKI, *New inequalities for some generalized Mathieu type series and the Riemann zeta function*, Journal of Mathematical Inequalities **12**, 1 (2018), 163–174.

[24] EDWARD CHARLES TITCHMARSH AND DAVID RODNEY HEATH-BROWN, *The theory of the Riemann zeta-function*, Oxford university press, 1986.

[25] ILAN VARDI, *Determinants of Laplacians and multiple gamma functions*, SIAM journal on mathematical analysis **19**, 2 (1988), 493–507.

[26] ANDRÉ VOROS, *Zeta functions over zeros of zeta functions*, vol. 8, Springer Science & Business Media, 2009.