

OPERATOR INEQUALITIES FOR h -CONVEX FUNCTIONS WITH APPLICATIONS

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Abstract. In this paper, we generalize the operator version of Jensen's inequality and the converse one for the class of h -convex functions. We extend the Hermite-Hadamard's type inequality and a multiple operator version of Jensen's inequality for this class of functions. We also provide a refinement of Jensen's inequality for convex functions. In particular, the operator h -convexity can be reduced to usual h -convexity in some sense and some results for the other classes of functions can be deduced by choosing an appropriate function h . The superiority of our results is that our results can recover some known results.

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