

SOME YANG–WANG–REN TYPE INEQUALITIES

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Abstract. In this paper, we shall present some weighted power mean inequalities: Let $a_{p,v}^{\sharp}b = ((1-v)a^p + vb^p)^{\frac{1}{p}}$. If $p \in [\frac{1}{2}, 1]$, $0 < v \leq \tau < 1$ and $m \in \mathbb{N}^+$, then we have

$$\frac{(a_{p,v}^{\sharp})^m - (a_v^{\sharp})^m}{(a_{p,\tau}^{\sharp})^m - (a_{\tau}^{\sharp})^m} \geq \frac{v(1-v)}{\tau(1-\tau)}$$

and

$$\frac{(a_{p,v}^{\sharp})^m - (a_v^{\sharp})^m}{(a_{p,\tau}^{\sharp})^m - (a_{\tau}^{\sharp})^m} \geq \frac{v(1-v)}{\tau(1-\tau)}$$

for $0 < b \leq a$; and the inequalities are reversed for $b \geq a > 0$. As applications, we obtain some inequalities for operator and determinant.

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