

BERNSTEIN–DOETSCH TYPE THEOREMS FOR DECREASING JENSEN m -CONVEX FUNCTIONS

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Abstract. The main purpose of this paper is to generalize the Bernstein-Doetsch theorem to the setting of the decreasing Jensen m -convex functions for $c_m \in \mathbb{N} \cap [2, \infty)$, where $c_m = \frac{m+1}{m}$. Also we ask: is this theorem still true for $c_m \in [2, \infty)$?

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