

A NEW VIEW ON SOME TWO-SIDED INEQUALITIES FOR FRAMES IN HILBERT SPACES

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Abstract. In this paper, we present a new approach to the proofs of several two-sided inequalities for frames in Hilbert spaces obtained recently by Xiang et al. from the point of view of function theory, which greatly simplifies the proving process. We also provide an improvement to two results of them on this topic. Finally, we establish a new two-sided inequality for frames in Hilbert spaces.

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REFERENCES

- [1] R. BALAN, P. G. CASAZZA, D. EDIDIN AND G. KUTYNIOK, *A new identity for Parseval frames*, Proc. Am. Math. Soc. **135**, 4 (2007), 1007–1015.
- [2] P. BALAZS, N. HOLIGHAUS, T. NECCIARI AND D. T. STOEVA, *Frame theory for signal processing in psychoacoustics*, in: R. Balan, J. Benedetto, W. Czaja, M. Dellatorre and K. Okoudjou (Eds.), Excursions in Harmonic Analysis, vol. 5, Appl. Numer. Harmon. Anal., Birkhäuser, Cham, 2017, pp. 225–268.
- [3] O. CHRISTENSEN, *An introduction to frames and Riesz bases*, Birkhäuser, Berlin (2016).
- [4] F. DAI, *Characterizations of function spaces on the sphere using frames*, Trans. Am. Math. Soc. **359**, 2 (2006), 567–589.
- [5] I. DAUBECHIES, A. GROSSMANN AND Y. MEYER, *Painless nonorthogonal expansions*, J. Math. Phys. **27**, 5 (1986), 1271–1283.
- [6] R. J. DUFFIN AND A. C. SCHAEFFER, *A class of nonharmonic Fourier series*, Trans. Am. Math. Soc. **72**, 2 (1952), 341–366.
- [7] Y. L. FU AND W. ZHANG, *Some new inequalities for dual continuous g-frames*, Mathematics **7**, 8 (2019), Article 662, 15pp.
- [8] P. GÄVRUȚA, *On some identities and inequalities for frames in Hilbert spaces*, J. Math. Anal. Appl. **321**, 1 (2006), 469–478.
- [9] D. W. LI AND J. S. LENG, *On some new inequalities for fusion frames in Hilbert spaces*, Math. Inequal. Appl. **20**, 3 (2017), 889–900.
- [10] D. W. LI AND J. S. LENG, *On some new inequalities for continuous fusion frames in Hilbert spaces*, Mediterr. J. Math. **15**, 4 (2018), Article 173, 15pp.
- [11] C. POON, *A consistent and stable approach to generalized sampling*, J. Fourier Anal. Appl. **20**, 5 (2014), 985–1019.
- [12] A. PORIA, *Some identities and inequalities for Hilbert-Schmidt frames*, Mediterr. J. Math. **14**, 2 (2017), Article 59, 14pp.
- [13] W. SUN, *Asymptotic properties of Gabor frame operators as sampling density tends to infinity*, J. Funct. Anal. **258**, 3 (2010), 913–932.
- [14] Z. Q. XIANG, *New types of inequalities for fusion frames*, J. Math. Inequal. **11**, 1 (2017), 291–299.
- [15] Z. Q. XIANG, C. X. LIN AND X. C. XIAO, *Frame inequalities in Hilbert spaces: two sided inequalities with new structures*, J. Math. Inequal. **18**, 2 (2024), 477–488.
- [16] W. ZHANG AND Y. Z. LI, *New inequalities and erasures for continuous g-frames*, Math. Rep. **20**, 3 (2018), 263–278.