

THE EXPONENTIAL FORM OF THE FRANK MATRIX

EFRUZ ÖZLEM MERSİN

Abstract. This paper focuses on the exponential form of the Frank matrix, one of the popular test matrices because of its well-conditioned and ill-conditioned eigenvalues. We explore the key properties of the matrix a^{F_n} for $a > 1$, highlighting its unique spectral characteristics, especially concerning its eigenvalues. By employing various mathematical techniques, including Sturm's Theorem, we establish that all eigenvalues of the matrix are real, positive and distinct. Furthermore, we provide a detailed characterization of its determinant, inverse, characteristic polynomial, LU decomposition, and several norms.

Mathematics subject classification (2020): 11C20, 15A16, 15A18, 15A42, 15B99.

Keywords and phrases: Frank matrix, matrix exponential, Sturm's Theorem, eigenvalue.

REFERENCES

- [1] M., ANĐELIĆ, C. M. DA FONSECA, C. KIZILATEŞ AND N. TERZIOĞLU, *r-min and r-max matrices with harmonic higher order Gauss Fibonacci numbers entries*, Journal of Applied Mathematics and Computing **71** (2025), 7437–7461.
- [2] C. M. DA FONSECA, C. KIZILATEŞ AND N. TERZIOĞLU, *A new generalization of min and max matrices and their reciprocals counterparts*, Filomat **38** (2024), 421–435.
- [3] P. J. EBERLEIN, *A note on the matrices denoted B_n^** , SIAM Journal on Applied Mathematics **20** (1971), 87–92.
- [4] W. L. FRANK, *Computing eigenvalues of complex matrices by determinant evaluation and by methods of Danilewski and Wielandt*, Journal of the Society for Industrial and Applied Mathematics **6** (1958), 378–392.
- [5] H. GÖKBAŞ, *Some properties of the generalized max Frank matrices*, AIMS Mathematics **9** (2024), 26826–26835.
- [6] L. GREENBERG, *Sturm sequences for nonlinear eigenvalue problems*, SIAM Journal of Mathematical Analysis **20** (1989), 182–199.
- [7] J. F. HAKE, *A remark on Frank matrices*, Computing (Wien. Print), **35** (1985), 375–379.
- [8] N. J. HIGHAM, *The power of bidiagonal matrices*, Electronic Journal of Linear Algebra **40** (2024), 453–474.
- [9] R. A. HORN AND C. R. JOHNSON, *Matrix analysis*, Cambridge University Press, 1985.
- [10] E. ISAACSON AND H. KELLER, *Analysis of numerical methods*, John Wiley, New York, 2nd edition, 1966.
- [11] C. KIZILATEŞ, *New families of horadam numbers associated with finite operators and their applications*, Mathematical Methods in the Applied Sciences **44** (2021), 14371–14381.
- [12] C. KIZILATEŞ AND N. TERZIOĞLU, *On r-min and r-max matrices*, Journal of Applied Mathematics and Computing **68** (2022), 4559–4588.
- [13] C. KIZILATEŞ, W. S. DU, AND F. QI, *Several determinantal expressions of generalized tribonacci polynomials and sequences*, Tamkang Journal of Mathematics **53** (2022), 277–291.
- [14] E. Ö. MERSİN, M. BAŞI, AND A. D. MADEN, *Some properties of generalized Frank matrices*, Mathematical Sciences and Applications E-Notes **8** (2020), 170–177.
- [15] E. Ö. MERSİN AND M. BAŞI, *Sturm theorem for the generalized Frank matrix*, Hacettepe Journal of Mathematics and Statistics **50** (2021), 1002–1011.

- [16] E. Ö. MERSİN AND M. BAŞI, *Bounds for the maximum eigenvalues of the Fibonacci-Frank and Lucas-Frank matrices*, Communications Faculty of Sciences University of Ankara Series A1 Mathematics and Statistics **73** (2024), 420–436.
- [17] A. MOSTOWSKI AND M. STARK, *Introduction to higher algebra*, Pergamon, Elsevier, 1964.
- [18] S. H. J. PETROUDI AND B. PIROUZ, *A particular matrix, its inversion and some norms*, Applied and Computational Mathematics **4** (2015), 47–52.
- [19] S. J. PETROUDI AND B. PIROUZ, *A note on Hadamard inverse and Hadamard exponential of a matrix with Fibonacci numbers*, Proceedings of the 7th National Conference on Mathematics, (2015), 28–29.
- [20] S. H. J. PETROUDI, M. PIROUZ, M. AKBIYIK, AND F. YILMAZ, *Some special matrices with harmonic numbers*, Konuralp Journal of Mathematics **10** (2022), 188–196.
- [21] K. PRASAD AND M. KUMARI, *Some new properties of Frank matrices with entries Mersenne numbers*, National Academy Science Letters, (2024), 1–7.
- [22] E. POLATLI, *On some properties of a generalized min matrix*, AIMS Mathematics **8** (2023), 26199–26212.
- [23] B. SHI, *A particular matrix with exponential form, its inversion and some norms*, AIMS Mathematics **7** (2022), 8224–8234.
- [24] B. SHI AND C. KIZILATEŞ, *A new generalization of the Frank matrix and its some properties*, Computational and Applied Mathematics **43** (2024), 19.
- [25] B. SHI AND C. KIZILATEŞ, *Geometric r -frank matrix: some properties and applications*, Computational and Applied Mathematics **44** (2025), 1–21.
- [26] J. STOER AND R. BULIRSCH, *Introduction to numerical analysis*, Springer-Verlag, New York, 2002.
- [27] C. STURM, *Analyse d'un Mémoire sur la résolution des équations numériques*, In Collected works of Charles François Sturm, Birkhäuser, Basel, (2009), 323–326.
- [28] C. STURM, *Extrait d'un Mémoire sur L'intégration d'un système d'équations différentielles linéaires, présenté à l'Académie des sciences*, In Collected works of Charles François Sturm, Birkhäuser, Basel, (2009), 334–342.
- [29] C. STURM, *Mémoire sur la résolution des équations numériques*, In Collected works of Charles François Sturm, Birkhäuser, Basel, (2009), 345–390.
- [30] J. M. VARAH, *A generalization of the Frank matrix*, SIAM Journal on Scientific and Statistical Computing **7** (1986), 835–839.