

UPPER BOUNDS FOR ERDÉLYI'S MULTIVARIATE LAGUERRE POLYNOMIALS

MIN-JIE LUO* AND RAVINDER KRISHNA RAINA

Abstract. We establish in this paper two inequalities for the multivariate Laguerre polynomials introduced and studied by Arthur Erdélyi [Sitzungsber. Akad. Wiss. Wien, Math.-Naturw. Kl., Abt. IIa 146 (1937), 431–467]. These inequalities generalize the well-known Szegő's inequality for the Laguerre polynomials $L_n^{(\alpha)}(x)$. We also mention briefly few insightful remarks giving a comparative analysis concerning the upper bounds of the derived inequalities in the concluding section.

Mathematics subject classification (2020): 33C45, 33E50.

Keywords and phrases: Laguerre polynomials, multivariate Laguerre polynomials.

REFERENCES

- [1] R. AKTAŞ, E. ERKUŞ-DUMAN, *The Laguerre polynomials in several variables*, Math. Slovaca **63** (2) (2013), 531–544.
- [2] B. C. CARLSON, *Special Functions of Applied Mathematics*, Academic Press, New York, 1977.
- [3] L. CARLITZ, *Bilinear generating functions for Laguerre and Lauricella polynomials in several variables*, Rend. Sem. Mat. Univ. Padova **43**, (1970) 269–276.
- [4] A. M. CHAK, *Some generalizations of Laguerre polynomials, II*, Mat. Vesnik (7) (22) (1970), 14–18.
- [5] A. N. DEMENIN, *On a class of orthogonal polynomials in several variables*, Ukr. Math. J. **23** (1971), 324–328.
- [6] A. ERDÉLYI, *Beitrag zur Theorie der konfluenten hypergeometrischen Funktionen von mehreren Veränderlichen*, Sitzungsber. Akad. Wiss. Wien, Math.-Naturw. Kl., Abt. IIa **146** (1937), 431–467.
- [7] R. A. C. FERREIRA, *Szegő's inequality revisited*, J. Math. Anal. Appl. **549** (2025), 129505.
- [8] G. H. HARDY, *Notes on special systems of orthogonal functions (I): The boundedness of the generalized Laguerre system*, J. London math. Soc. **14** (1939), 34–36.
- [9] Z. LEWANDOWSKI, J. SZYNAL, *An upper bound for the Laguerre polynomials*, J. Comput. Appl. Math. **99** (1–2) (1998), 529–533.
- [10] S.-J. LIU, S.-D. LIN, H.-C. LU, H. M. SRIVASTAVA, *Linearization of the products of the generalized Lauricella polynomials and the multivariate Laguerre polynomials*, Stud. Sci. Math. Hung. **50** (3) (2013), 373–391.
- [11] E. R. LOVE, *Inequalities for Laguerre functions*, J. Inequal. Appl. **1** (3) (1997), 293–299.
- [12] M.-J. LUO, R. K. RAINA, *Two inequalities for the associated Pollaczek polynomials*, J. Inequal. Appl. **2020** (2020), Paper No. 9, 13 p.
- [13] M. MICHALSKA, J. SZYNAL, *A new bound for the Laguerre polynomials*, J. Comput. Appl. Math. **133** (1–2) (2001), 489–493.
- [14] F. W. J. OLVER, D. W. LOZIER, R. F. BOISVERT, C. W. CLARK (Eds.), *NIST Handbook of Mathematical Functions*, Cambridge University Press, New York, 2010.
- [15] R. PANDA, *Some multiple series transformation*, Jñānābha, Sect. A, **4** (1974), 165–168.
- [16] R. PEMANTLE, M. C. WILSON, *Analytic Combinatorics in Several Variables*, Cambridge Studies in Advanced Mathematics, 140. Cambridge University Press, Cambridge, 2013.
- [17] R. PETERSEN, H. SKOVGAARD, *On an equiconvergence theorem for Laguerre series*, Mat. Tidsskr. B **1952**, (1952), 14–27.

- [18] T. K. POGÁNY, H. M. SRIVASTAVA, *Some improvements over Love's inequality for the Laguerre function*, *Integral Transforms Spec. Funct.* **18** (5) (2007), 351–358.
- [19] P. G. ROONEY, *An inequality for generalized Laguerre polynomials*, *C. R. Math. Rep. Acad. Sci. Canada* **6** (1984), 361–364.
- [20] P. G. ROONEY, *Further inequalities for generalized Laguerre polynomials*, *C. R. Math. Rep. Acad. Sci. Canada* **7** (1985), 273–275.
- [21] H. M. SRIVASTAVA, *Some bounding inequalities for the Jacobi and related functions*, *Banach J. Math. Anal.* **1** (1) (2007), 131–138.
- [22] H. M. SRIVASTAVA, P. W. KARLSSON, *Multiple Gaussian Hypergeometric Series*, Ellis Horwood Ltd., Chichester, 1985.
- [23] H. M. SRIVASTAVA, A. W. NIUKKANEN, *Some Clebsch-Gordan type linearization relations and associated families of Dirichlet integrals*, *Math. Comput. Modelling* **37** (3–4) (2003), 245–250.
- [24] G. SZEGŐ, *Ein Beitrag zur Theorie der Polynome von Laguerre und Jacobi*, *Math. Z.* **1** (4) (1918), 341–356.
- [25] G. N. WATSON, *Another note on Laguerre polynomials*, *J. Lond. Math. Soc.* **14** (1939), 19–22.