

FORMABLE INTEGRAL TRANSFORM AND ULAM STABILITY OF n TH ORDER LINEAR DIFFERENTIAL EQUATIONS

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Abstract. In this current manuscript, we prove the Hyers-Ulam stability of homogeneous and nonhomogeneous linear differential equations by using a new transform called formable integral transform. Further, we present counter-examples of formable integral transforms for the Hyers-Ulam stability approach.

Mathematics subject classification (2020): 26D10, 44A10, 39B82, 34A40, 39A30, 34K20.

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