

INTEGRAL OPERATORS AND THEIR COMMUTATORS IN VARIABLE EXPONENT GENERALIZED WEIGHTED MORREY–GULIYEV SPACES

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Abstract. In this paper we consider the variable exponent generalized weighted Morrey-Guliyev spaces $GM^{p(\cdot),\theta(\cdot),\omega(\cdot),\varphi}(\Omega)$ with variable exponents $p(x)$, $\theta(r)$ and a general function $\omega(x,r)$ defining the Morrey-type norm. In case of unbounded sets $\Omega \subseteq \mathbb{R}^n$ we prove the boundedness of Hardy-Littlewood maximal operator, generalized fractional integral operators and Calderón-Zygmund singular integral operators in weighted Morrey-Guliyev spaces. Furthermore, we investigate the boundedness of the commutators of abovementioned integral operators in weighted Morrey-Guliyev spaces.

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