

DIFFERENCE OF DIFFERENTIATION COMPOSITION OPERATORS FROM THE BESOV SPACE B^1 TO BLOCH-TYPE SPACES AND LITTLE BLOCH-TYPE SPACES

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Abstract. We characterize bounded and compact differences of differentiation composition operators from the Besov space B^1 to Bloch-type spaces utilizing the difference of the values of derivatives of the conformal automorphisms of the unit disk at the respective symbols of the these operators. We also derive asymptotic estimates for the operator norms, providing significant insights into their behavior. In addition we characterize boundedness and compactness of $C_\varphi D^n - C_\psi D^n$ from the minimal Möbius invariant space B^1 to little Bloch-type spaces. Our characterization relies on the analysis of the derivatives of conformal automorphisms of \mathbb{D} , revealing intricate relationships between conformal automorphisms of \mathbb{D} and differentiation composition operators between these spaces.

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