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POWER–TYPE INEQUALITIES IN THE FRAMEWORK OF p–CONVEXITY WITH APPLICATIONS TO MATRIX INEQUALITIES

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Abstract. Convexity, log-convexity, and p-convexity are fundamental tools in analysis and matrix theory, serving as powerful techniques for deriving functional and operator inequalities. In this paper, we establish several new refinements and generalizations of classical convex inequalities, extending earlier results of Sababheh, Alzer, Heinz, and Young-type. Our approach is based on convexity-inspired factorizations and power-type extensions, which allow us to derive sharper inequalities for p-convex, convex and log-convex functions. Applications are provided to a wide range of mathematical means, including arithmetic, geometric, harmonic, power, and Heinz means, where we obtain improved bounds and new inequalities valid for arbitrary integer powers. Furthermore, we apply these results to matrix analysis, yielding refinements of determinant inequalities, inequalities for unitarily invariant norms, and Hölder-type inequalities. Additional applications are given to generalized numerical radius inequalities for operators. These improvements not only unify and extend several known results but also provide new insights into the interplay between convexity, p-convexity, matrix inequalities, and operator theory.

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