

## NOTE ON NORM ESTIMATE OF A LINEAR OPERATOR FROM THE CLASSICAL WEIGHTED BERGMAN SPACE TO THE $m$ TH WEIGHTED-TYPE SPACE

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*Abstract.* We give an upper and a lower bound for the norm of a linear operator of a polynomial differentiation composition type from the classical weighted Bergman space to the  $m$ th weighted-type space and the little  $m$ th weighted-type space on the open unit disk in the complex plane, which were introduced by us about twenty years ago. We also give some sufficient conditions and some necessary conditions for the boundedness of the operator.

*Mathematics subject classification (2020):* Primary 47B38.

*Keywords and phrases:* Operator norm, polynomial differentiation composition operator, classical weighted Bergman space,  $m$ th weighted-type space, bounded operator.

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