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SOME NEW IMPROVEMENTS FOR MULTIPLE–TERM VERSIONS OF ALZER–FONSECA–KOVAČEC'S TYPE INEQUALITIES

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Abstract. In this paper, we build on the recent advancements in the theory of weakly submajorizations established by D. Q. Huy et al. [Linear Algebra Appl., doi.org/10.1016/j.laa.2025.10.004] to present innovative real power-type and multiple-term refinements of Alzer-Fonseca-Kovačec-type inequalities. These refinements are accompanied by notable applications, including their extensions to operator versions, unitarily invariant norms, and matrix determinants.

Mathematics subject classification (2020): 26D15, 47A63, 15A60, 47A30, 15A39, 15B48.
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REFERENCES

[1] H. ALZER, C. FONSECA, A. KOVAČEC, *Young-type inequalities and their matrix analogues*, Linear Multilinear Algebra **63** (3) (2015), 622–635.
[2] T. ANDO, *Matrix Young inequality*, Oper. Theory Adv. Appl. **75** (1995), 33–38.
[3] D. CHOI, *Multiple-term refinements of Young type inequalities*, J. Math. **2016**, Art. ID 4346712, 11 pp.
[4] D. CHOI, M. KRNIĆ, J. PEČARIĆ, *Improved Jensen-type inequalities via linear interpolation and applications*, J. Math. Inequal. **11** (2) (2017), 301–322.
[5] D. CHOI, *A generalization of Young-type inequalities*, Math. Inequal. Appl. **21** (1) (2018), 99–106.
[6] D. Q. HUY, D. T. T. VAN, D. T. XINH, *Some generalizations of real power form for Young-type inequalities and their applications*, Linear Algebra Appl. **656** (2023), 368–384.
[7] D. Q. HUY, T. T. QUANG, D. T. T. VAN, *Multiple-term refinements and reverses of real power form for Young-type inequalities via weak submajorization type theorems*, Linear Algebra Appl. **729** (2026), 256–292.
[8] M. A. IGHACHANE, M. AKKOUCHI, E. H. BENABDI, *Further refinements of Alzer-Fonseca-Kovacec's inequalities and applications*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **115** (3) (2021), Art. 152, 14 pp.
[9] M. A. IGHACHANE, M. AKKOUCHI, *Further refinements of Young's type inequality for positive linear maps*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **115** (52) (2021).
[10] M. A. IGHACHANE, *Multiple-term refinements of Alzer-Fonseca-Kovacec inequalities*, Rocky Mountain J. Math. **52** (6) (2022), 2053–2070.
[11] M. A. IGHACHANE, Z. TAKI, M. BOUCHANGOUR, *An improvement of Alzer-Fonseca-Kovacec's type inequalities with applications*, Filomat **37** (22) (2023), 7383–7399.
[12] M. A. IGHACHANE, M. AKKOUCHI, M. SABABHEH, *Power inequalities for log-convex functions with applications*, Filomat **37** (13) (2023), 4425–4441.
[13] M. A. IGHACHANE, M. AKKOUCHI, M. SABABHEH, *New inequalities for positive convex functions*, J. Appl. Anal. Comput. **14** (2) (2024), 703–716.
[14] M. A. IGHACHANE, E. H. BENABDI, M. AKKOUCHI, M. SABABHEH, *Advanced refinements of matrix mean inequalities via log-convexity*, Bull. Malays. Math. Sci. Soc. **48**, Art. 97 (2025).
[15] F. KITTANEH, Y. MANASRAH, *Improved Young and Heinz inequalities for matrices*, J. Math. Anal. Appl. **361** (1) (2010), 262–269.

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[16] F. KITTANEH, Y. MANASRAH, *Reverse Young and Heinz inequalities for matrices*, Linear Multilinear Algebra **59** (9) (2011), 1031–1037.
[17] P. KÓRUS, *A refinement of Young's inequality*, Acta Math. Hungar. **153** (2017), 430–435.
[18] H. KOSAKI, *Arithmetic-geometric mean and related inequalities for operators*, J. Funct. Anal. **156** (1998), 429–451.
[19] Y. MANASRAH, F. KITTANEH, *A generalization of two refined Young inequalities*, Positivity **19** (4) (2015), 757–768.
[20] Y. REN, *Some results of Young-type inequalities*, Rev. R. Acad. Cienc. Exactas Fís. Nat. Ser. A Mat. RACSAM **114**(3) (2020), Art. 143, 10 pp.
[21] Y. REN, P. LI, *Some results of reverses Young's inequalities*, Filomat **36** (8) (2022), 2541–2550.
[22] M. SABABHEH, D. CHOI, *A complete refinement of Young's inequality*, J. Math. Anal. Appl. **440** (1) (2016), 379–393.
[23] M. SABABHEH, *Convexity and matrix means*, Linear Algebra Appl. **506** (2016), 588–602.
[24] M. SABABHEH, M. S. MOSLEHIAN, *Advanced refinements of Young and Heinz inequalities*, J. Number Theory **172** (2017), 178–199.
[25] M. SABABHEH, *Graph indices via the AM-GM inequality*, Discrete Appl. Math. **230** (2017), 100–111.
[26] C. YANG, Z. WANG, *Some new improvements of Young's inequalities*, J. Math. Inequal. **17** (1) (2023), 205–217.
[27] J. ZHAO, *Further refinements and generalizations of the Young's and its reverse inequalities*, Bull. Malays. Math. Sci. Soc. **46** (2) (2023), Art. 52, 21 pp.