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COMPLETE CONVERGENCE AND COMPLETE MOMENT CONVERGENCE FOR WEIGHTED SUMS OF MARTINGALE DIFFERENCE RANDOM VECTORS

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Abstract. Let $\{X_{ni}, \mathscr{F}_{ni}; 1 \leqslant i \leqslant n, n \geqslant 1\}$ be an array of $d \times 1$ martingale difference random vectors weakly summable dominated by a random vector X concerning the array $\{A_{ni}, 1 \leqslant i \leqslant n, n \geqslant 1\}$ of $m \times d$ matrices of real numbers. Under almost optimal conditions, we proved that for any $\varepsilon > 0$,

$$\sum_{n=1}^{\infty} n^{-1} P\left(\max_{1 \leqslant m \leqslant n} \left\| \sum_{i=1}^{m} A_{ni} X_i \right\| > \varepsilon n^{1/\alpha} \log^{1/\gamma} n \right) < \infty,$$

and

$$\sum_{n=1}^{\infty} n^{-1} E\left(b_n^{-1/\alpha} \log^{-1/\gamma} n \max_{1 \leqslant m \leqslant n} \left\| \sum_{i=1}^{m} A_{ni} X_{ni} \right\| - \varepsilon \right)^{+} < \infty.$$

The main results provide a multi-dimensional extension of some corresponding ones in the literature, improving upon the existing one-dimensional theory. Moreover, by imposing a slightly stronger assumption on the weight matrices, we also obtain the desired results under a weaker moment condition and a more flexible range for γ.

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