

WILKER TYPE INEQUALITIES WITH TWO-PARAMETERIZED QUADRATIC POWER EXPONENTIAL FUNCTIONS

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Abstract. In this paper, we present new Wilker type inequality with power exponential functions as follows. The inequality

$$\left(\frac{\sin x}{x}\right)^{2+ax^2} + \left(\frac{\tan x}{x}\right)^{1-bx^2} > 2$$

holds for $0 < x < \frac{\pi}{2}$ and (a, b) in some closed subset $D \subset \mathbb{R}^2$. This is the first study to assign a function of x to the exponential part of $\frac{\sin x}{x}$ and $\frac{\tan x}{x}$ so that the Wilker inequality holds.

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