

A GENERALIZATION OF YOUNG'S ℓ^p INEQUALITY

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Abstract. We show that, for positive real numbers with $a \geq 1 + \sum_i^N \alpha_i$, the function

$$\frac{t^a}{\prod_{i=1}^N x_i^{\alpha_i}}$$

has a convex conjugate of the same form and so, in particular, obtain a clean proof that f is convex.

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