

DELAY–DIFFERENCE EQUATIONS WITH PERIODIC COEFFICIENTS: SHARP RESULTS IN OSCILLATION THEORY

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Abstract. The following aspects of oscillation theory for Delay-difference equation

$$x(n+1) - x(n) + b(n)x(n-k) = 0, \quad k \in \mathbf{N}, b(n) \geq 0, n \geq n_0 \quad (1)$$

are investigated:

1. Two following facts are equivalent:
 - a) There exists at least one non-oscillatory solution of Eq. (1).
 - b) There is no regularly (slowly) oscillatory solution of Eq. (1).
2. An explicit condition for oscillation of all solutions of Eq. (1), in which $b(n) \geq a(n) \geq 0$, $a(n+r) = a(n) \forall n$, is elaborated. In the particular case $b(n) = a(n)$, $\forall n$ it turns to the *sufficient and necessary condition*. For some particular cases $\{r = k\}$; $\{r = k+1\}$; $\{k = 1, r = 3 \text{ or } 4\}$; $\{r = 2\}$ this condition is formulated in a recognisable form in terms of coefficients.

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