

## INEQUALITIES FOR THE MINIMAL EIGENVALUE OF THE LAPLACIAN IN AN ANNULUS

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*Abstract.* We discuss the behavior of the minimal eigenvalue  $\lambda$  of the Dirichlet Laplacian in the domain  $D_1 \setminus D_2 := D$  (an annulus) where  $D_1$  is a circular disc and  $D_2 \subset D_1$  is a smaller circular disc. It is conjectured that the minimal eigenvalue  $\lambda$  has a maximum value when  $D_2$  is a concentric disc. If  $h$  is a displacement of the center of the disc  $D_2$  and  $\lambda(h)$  is the corresponding minimal eigenvalue, then  $\frac{d\lambda(h)}{dh} < 0$  so that  $\lambda(h)$  is minimal when  $\partial D_2$  touches  $\partial D_1$ , where  $\partial D$  is the boundary of  $D$ . Numerical results are given to back the conjecture. Upper and lower bounds are given for  $\lambda(h)$ .

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