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INEQUALITIES FOR BRACHISTOCHRONE

A. G. RAMM

Abstract. Let $A = (0,1)$ and $B = (b,0)$ be the initial and final points, and $y = y(x)$ joins A with AB, $y'' \geqslant 0$, $0 \leqslant y(x) \leqslant y_0(x)$, where $y_0(x)$ is the straight line joining A and B. Denote by S the set of such $y(x)$. Let P be the polygon consisting of two segments of straight lines: $A0$ and OB, where $0 = (0,0)$ is the origin. Up to a constant factor depending on the choice of units, the time needed for a particle to get from A to B along $y(x)$ in the gravitational field is $T(y) = \int_0^b \frac{\sqrt{1+y'^2}}{\sqrt{1-y}}\, dx$. Let $T_0 := T(y_0) = 2\sqrt{1+b^2}$, $T_P := T(P) = 2+b$. It is conjectured that:

1) if $0 < b < \frac{4}{3}$ then $T(y_{br}) \leqslant T(y) < T_P$, $y \in S$,
2) if $\frac{4}{3} \leqslant b \leqslant \frac{\pi}{2}$ then $T(y_{br}) \leqslant T(y) \leqslant T_0$, $y \in S$,
3) if $b > \frac{\pi}{2}$ then $T(P_{br}) < T(y) \leqslant T_0$, $y \in S$

where $y_{br} = y_{br}(x) \in S$ is the classical brachistochrone curve. For $b > \frac{\pi}{2}$ this curve probably degenerates into P_{br}, the brachistochrone curve which joins A and $(\pi/2, 0)$ and the straight line joining $(\pi/2, 0)$ and $(b, 0)$.

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