

GROWTH OF MAXIMUM MODULUS OF RATIONAL FUNCTIONS WITH PRESCRIBED POLES

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Abstract. In this paper we prove several sharp growth estimates for rational functions with prescribed poles in the Chebyshev norm on the unit circle in the complex plane. In particular, our results generalize and sharpen certain polynomial inequalities due to Rahman, Ankeny and Rivlin for such rational functions with restricted zeros.

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