

INEQUALITIES FOR THE ZEROS OF THE ASSOCIATED ULTRASPHERICAL POLYNOMIALS

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Abstract. Using a functional analytic method based on the three-term recurrence relation of orthogonal polynomials, we study the monotonicity of the zeros of the associated ultraspherical (or Gegenbauer) polynomials, and we give some inequalities for the largest zeros.

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