

SOME INEQUALITIES CONSTRUCTED BY TCHEBYSHEFF'S INTEGRAL INEQUALITY

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Abstract. In the article, using Tchebysheff's integral inequality, the authors refine Conte's inequality and some estimates concerning the probability integral which are related to the Mills' ratio, form several inequalities of trigonometric functions, obtain some integral inequalities and estimates of definite integrals related to $\sin x/x$, $x \in [0, \pi/2]$, and construct many miscellaneous inequalities.

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