

NEW PROOFS OF WEIGHTED POWER MEAN INEQUALITIES AND MONOTONICITY FOR GENERALIZED WEIGHTED MEAN VALUES

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Abstract. In the article, a new proof of the weighted power mean inequalities is given using Cauchy-Schwarz-Buniakowski's inequality, and another two simple and short proofs of monotonicity for the generalized weighted mean values with two parameters are showed.

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REFERENCES

- [1] H. ALZER, *A proof of the arithmetic mean-geometric mean inequality*, Amer. Math. Monthly **103** (1996), 585.
- [2] P. S. BULLEN, D. S. MITRINOVIĆ, AND P. M. VASIĆ, *Means and Their Inequalities*, D. Reidel Publ. Company, Dordrecht/Boston/Lancaster/Tokyo, 1988.
- [3] JI-CHANG KUANG, *Applied Inequalities*, 2nd edition, Hunan Education Press, Changsha, China, 1993. (Chinese)
- [4] E. B. LEACH AND M. C. SHOLANDER, *Extended mean values*, Amer. Math. Monthly **85** (1978), 84–90.
- [5] LÁSZLÓ LOSONCZI, *Inequalities for integral mean values*, J. Math. Anal. Appl. **61** (1977), 586–606.
- [6] D. S. MITRINOVIĆ, *Analytic Inequalities*, Springer-Verlag, Berlin, 1970.
- [7] D. S. MITRINOVIĆ, J. E. PEČARIĆ, AND A. M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, 1993.
- [8] J. PEČARIĆ, *Nejednakosti*, Element, Zagreb, 1996.
- [9] J. PEČARIĆ, FENG QI, V. ŠIMIĆ, AND SEN-LIN XU, *Refinements and extensions of an inequality*, III, J. Math. Anal. Appl. **227** (1998), no. 2, 439–448.
- [10] J. PEČARIĆ AND S. VAROŠANEC, *A new proof of the arithmetic mean-the geometric mean inequality*, J. Math. Anal. Appl. **215** (1997), 577–578.
- [11] FENG QI, *Generalized weighted mean values with two parameters*, Proc. Roy. Soc. London Ser. A **454** (1998), no. 1978, 2723–2732.
- [12] FENG QI, *On a two-parameter family of nonhomogeneous mean values*, Tamkang J. Math. **29** (1998), no. 2, 155–163.
- [13] FENG QI, *Generalized abstracted mean values*, Journal of Inequalities in Pure and Applied Mathematics **1** (2000), no. 1, Article 4. http://jipam.vu.edu.au/v1n1/013_99.html. RGMIA Research Report Collection **2** (1999), no. 5, Article 4. <http://rgmia.vu.edu.au/v2n5.html>.
- [14] FENG QI, *Logarithmic convexities of the extended mean values*, RGMIA Research Report Collection **2** (1999), no. 5, Article 5. <http://rgmia.vu.edu.au/v2n5.html>.
- [15] FENG QI AND QIU-MING LUO, *A simple proof of monotonicity for extended mean values*, J. Math. Anal. Appl. **224** (1998), 356–359.
- [16] FENG QI, JIA-QIANG MEI, AND SEN-LIN XU, *Other proofs of monotonicity for generalized weighted mean values*, RGMIA Research Report Collection **2** (1999), no. 4, Article 6. <http://rgmia.vu.edu.au/v2n4.html>.
- [17] FENG QI, SEN-LIN XU, AND LOKENATH DEBNATH, *A new proof of monotonicity for extended mean values*, Intern. J. Math. Math. Sci. **22** (1999), no. 2, 415–420.

- [18] FENG QI AND SEN-LIN XU, *Refinements and extensions of an inequality*, II, J. Math. Anal. Appl. **211** (1997), 616–620.
- [19] FENG QI AND SEN-LIN XU, *The function $(b^x - a^x)/x$: Inequalities and properties*, Proc. Amer. Math. Soc. **126** (1998), no. 11, 3355–3359.
- [20] FENG QI AND SHI-QIN ZHANG, *Note on monotonicity of generalized weighted mean values*, Proc. Roy. Soc. London Ser. A **455** (1999), no. 1989, 3259–3260.
- [21] DA-FENG XIA, SEN-LIN XU, AND FENG QI, *A proof of the arithmetic mean-geometric mean-harmonic mean inequalities*, RGMIA Research Report Collection **2** (1999), no. 1, Article 10, 99–102. <http://rgmia.vu.edu.au/v2n1.html>.
- [22] REN-ER YANG AND DONG-JI CAO, *Generalizations of the logarithmic mean*, J. Ningbo Univ. **2** (1989), no. 2, 105–108.