

RESULTS UNDER $\log A \geq \log B$ CAN BE DERIVED FROM ONES UNDER $A \geq B \geq 0$ BY UCHIYAMA'S METHOD – ASSOCIATED WITH FURUTA AND KANTOROVICH TYPE OPERATOR INEQUALITIES

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Abstract. In what follows, a capital letter means a bounded linear operator on a Hilbert space H . We discuss some parallelism between $A \geq B \geq 0$ and $\log A \geq \log B$ on generalized Furuta inequality and Kantorovich type inequalities. Precisely speaking, several results under the chaotic order $\log A \geq \log B$ on Furuta type inequalities and Kantorovich type inequalities can be both derived from ones under the usual order $A \geq B \geq 0$ by using Uchiyama's method [23].

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