

EXISTENCE OF SOLUTIONS FOR $2n^{\text{th}}$ ORDER NONLINEAR GENERALIZED STURM-LIOUVILLE BOUNDARY VALUE PROBLEMS

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Abstract. Higher order upper and lower solutions are used to establish the existence of solutions to $y^{(2n)} = f(t, y, y'', \dots, y^{(2n-2)})$, satisfying nonlinear boundary conditions, either of the form $g_i(y^{(2i-2)}(0), y^{(2i-1)}(0)) = 0$, $h_i(y^{(2i-2)}(1), y^{(2i-1)}(1)) = 0$, $1 \leq i \leq n$, or of the form $k_i(y^{(2i-2)}(0), y^{(2i-2)}(1)) = 0$, $\ell_i(y^{(2i-2)}(0), y^{(2i-2)}(1)) = 0$, $1 \leq i \leq n$.

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REFERENCES

- [1] R. P. AGARWAL AND P. J. Y. WONG, *Lidstone polynomials and boundary value problems*, Comp. and Math. Appl., **17**(1989), 1397–1421.
- [2] R. P. AGARWAL AND P. J. Y. WONG, *Eigenvalues of Lidstone boundary value problems*, Appl. Math. Comp., **104**(1999), 15–31.
- [3] R. P. AGARWAL AND P. J. Y. WONG, *Results and estimates on multiple solutions of Lidstone boundary value problems*, Acta Math. Hungarica, in press.
- [4] K. AKO, *Subfunctions for ordinary differential equations I*, J. Fac. Sci. Univ. Tokyo **9**(1965), 17–43.
- [5] K. AKO, *Subfunctions for ordinary differential equations II*, Funckialaj Ekvacioj **10**(1967), 145–162.
- [6] K. AKO, *Subfunctions for ordinary differential equations III*, Funckialaj Ekvacioj **11**(1968), 111–129.
- [7] C. F. BEARDS, “*Vibration Analysis with Applications to Control Systems*”, Edward Arnold, London, 1995.
- [8] E. DULÁCSKA, “*Soil Settlement Effects on Buildings*”, Developments in Geotechnical Engineering, **vol. 69**, Elsevier, Amsterdam, 1992.
- [9] J. M. DAVIS, P. W. ELOE AND J. HENDERSON, *Triple positive solutions and dependence on higher order derivatives*, J. Math. Anal. Appl. **237**(1999), 710–720.
- [10] J. M. DAVIS, L. H. ERBE AND J. HENDERSON, *Multiplicity of positive solutions for higher order Sturm-Liouville problems*, Rocky Mtn. J. Math., in press.
- [11] P. W. ELOE AND L. J. GRIMM, *Monotone iteration and Green’s functions for boundary value problems*, Proc. Amer. Math. Soc. **78**(1980), 533–538.
- [12] P. W. ELOE AND J. HENDERSON, *A boundary value problem for a system of ordinary differential equations with impulse effects*, Rocky Mtn. J. Math. **27**(1997), 785–799.
- [13] R. E. GAINES, *A priori bounds and upper and lower solutions for nonlinear second-order boundary value problems*, J. Differential Equations **12**(1972), 291–312.
- [14] J. HENDERSON AND H. B. THOMPSON, *Multiple symmetric positive solutions for a second order boundary value problem*, Proc. Amer. Math. Soc., **128** (2000), 2373–2379. press.
- [15] J. HENDERSON AND H. B. THOMPSON, *Existence of multiple solutions for second order boundary value problems*, J. Differential Equations, **166** (2000), 443–454.
- [16] S. H. HONG AND S. G. HU, *A monotone iterative method for higher-order boundary value problems*, Math. Appl. **12**(1999), 14–18.
- [17] L. K. JACKSON, *Subfunctions and second-order ordinary differential equations*, Advances in Math. **2**(1968), 307–363.

- [18] W. G. KELLEY, *Some existence theorems for n th-order boundary value problems*, J. Differential Equations **18**(1975), 158–169.
- [19] G. A. KLAASEN, *Differential inequalities and existence theorems for second and third order boundary value problems*, J. Differential Equations **10**(1971), 529–537.
- [20] J. L. MAWHIN, “*Topological Degree Methods in Nonlinear Boundary Value Problems*”, Regional Conference Series in Math., No. 40, American Mathematical Society, Providence, 1979.
- [21] L. MEIROVITCH, “*Dynamics and Control of Structures*”, Wiley, New York, 1990.
- [22] M. NAGUMO, *Über die Differentialgleichungen $y'' = f(x, y, y')$* , Proc. Phys.-Math. Soc. Japan **19**(1937), 861–866.
- [23] V. ŠEDA, *Two remarks on boundary value problems for ordinary differential equations*, J. Differential Equations **26**(1977), 278–290.
- [24] K. SCHMITT, *Boundary value problems and comparison theorems for ordinary differential equations*, SIAM J. Appl. Math. **26**(1976), 670–678.
- [25] H. B. THOMPSON, *Second order ordinary differential equations with fully nonlinear two point boundary conditions*, Pacific J. Math. **172**(1996), 255–276.
- [26] H. B. THOMPSON, *Second order ordinary differential equations with fully nonlinear two point boundary conditions II*, Pacific J. Math. **172**(1996), 279–297.