

ON MAXIMUM MODULUS OF POLYNOMIALS AND RELATED ENTIRE FUNCTIONS WITH RESTRICTED ZEROS

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Abstract. Let $p(z) = \sum_{v=0}^n a_v z^v$ be a polynomial of degree n , and $M(p, r) = \max_{|z|=r} |p(z)|$. As a generalization of a well known result of Rivlin [5] and some other results ([1], [4]) in this direction, Jain [2] besides proving some other results, also proved that if the polynomial $p(z) = a_0 + \sum_{v=m}^n a_v z^v$ has no zeros in $|z| < k$, $k > 0$, then for $0 \leq r < R \leq k$, $M(p, r) \geq \left(\frac{r^m + k^m}{R^m + k^m} \right)^{n/m} M(p, R)$. In this paper we present very simple proofs of this and other results of Jain [2].

Mathematics subject classification (2000): 30C10, 30C80, 30D15, 26C10, 26D10.

Key words and phrases: Inequalities in the complex domain, polynomials, maximum modulus principle, special classes of entire functions and growth estimates.

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