

ON THE MINIMUM NUMBER OF DISTINCT EIGENVALUES
FOR A SYMMETRIC MATRIX
WHOSE GRAPH IS A GIVEN TREE

ANTÓNIO LEAL-DUARTE AND CHARLES R. JOHNSON

Abstract. It is shown that for any tree T the minimum number of distinct eigenvalues of an Hermitian matrix whose graph is T (diagonal entries free) is at least the number of vertices in a longest path of T . This is another step toward the general problem of characterizing the possible multiplicities for a given graph. Related observations are made and the result facilitates a table of multiplicities for trees on fewer than 8 vertices.

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