

INTERPOLATION POLYNOMIALS AND INEQUALITIES FOR CONVEX FUNCTIONS OF HIGHER ORDER

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Abstract. In this paper we prove several inequalities for convex function of a higher order. Generalizations of Hadamard's inequalities and the conversion of Jensen's inequality for (n) – convex function and with conditions on the regular, real (signed) Borel measure are presented by Lidstone's and Hermite's interpolating polynomials. As a discrete form we also obtain a generalization of Perović's inequality, i.e. Giaccardi's inequality. The Abel-Gontscharoff interpolating polynomial with two-point right focal conditions leads us to an inequality like converse of Jensen inequality for a regular, signed measure and, as a consequence, to some inequalities related with Hadamard's and Petrović's inequalities.

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