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GOLDEN–THOMPSON TYPE INEQUALITIES RELATED TO A GEOMETRIC MEAN VIA SPECHT'S RATIO

MASATOSHI FUJII, YUKI SEO AND MASARU TOMINAGA

Abstract. We prove a Golden-Thompson type inequality via Specht's ratio: Let H and K be selfadjoint operators on a Hilbert space H satisfying $MI \geqslant H, K \geqslant mI$ for some scalar $M > m$. Then

$$M_h(1)\left((1-\lambda)e^{tH}+\lambda e^{tK}\right)^{\frac{1}{t}} \geqslant e^{(1-\lambda)H+\lambda K} \geqslant M_h(1)^{-1}M_h(t)^{-\frac{1}{t}}\left((1-\lambda)e^{tH}+\lambda e^{tK}\right)^{\frac{1}{t}}$$

holds for all $t > 0$ and $0 \leqslant \lambda \leqslant 1$, where $h = e^{M-m}$ and (generalized) Specht's ratio $M_h(t)$ is defined for $h > 0$ as

$$M_h(t) = \frac{(h^t-1)h^{\frac{t}{h^t-1}}}{e\log h^t} \quad (h \neq 1) \quad \text{and} \quad M_1(1) = 1.$$

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