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A LAGRANGIAN DUAL METHOD FOR SOLVING VARIATIONAL INEQUALITIES

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Abstract. In this paper we consider a variational inequality problem (VIP) defined by a maximal monotone operator and a feasible set defined by convex inequality constraints and bounds on the variables. A Lagrangian dual method for solving this problem is presented and its convergence is proved.

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