

HEINZ–KATO–FURUTA–TYPE INEQUALITIES WITH BOUNDS AND EQUALITY CONDITIONS

C. - S. LIN

Abstract. In this article we present several new generalized Heinz-Kato-Furuta-type inequalities. The main tools used here are the Cauchy-Schwarz inequality, basic properties of the polar decomposition of an operator, and, of course, the Furuta inequality. Such considerations enable us to obtain inequalities very easily. The bounds and equality conditions for generalized Heinz-Kato-Furuta-type inequalities are given.

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