

DIFFERENCE DERIVED FROM WEIGHTED HÖLDER'S INEQUALITY

SAICHI IZUMINO, JOSIP PEČARIĆ AND MASARU TOMINAGA

Abstract. We give the maximum of the difference

$$D_p(a, b; w) := \left(\sum_{k=1}^n w_k a_k^p \right)^{\frac{1}{p}} \left(\sum_{k=1}^n w_k b_k^q \right)^{\frac{1}{q}} - \sum_{k=1}^n w_k a_k b_k$$

derived from a weighted Hölder's inequality for $p, q > 1$, $p^{-1} + q^{-1} = 1$ and for positive n -tuples $a := (a_1, \dots, a_n)$, $b := (b_1, \dots, b_n)$ and a weight $w := (w_1, \dots, w_n)$ under certain conditions. The discussion in this note is simpler than our previous ones. It comes from the arrangement of a given weight and a linearization of $D_p(a, b; w)$ via Young's inequality. As a consequence, we give a , b and w which attain the maximum.

Mathematics subject classification (2000): 26D15.

Key words and phrases: Hölder's inequality, convex function, Ozeki's inequality.

REFERENCES

- [1] S. IZUMINO, *Ozeki's method on Hölder's inequality*, Math. Japon., **50**, (1999), 41–55.
- [2] S. IZUMINO, H. MORI AND Y. SEO, *On Ozeki's inequality*, J. Inequal. Appl., **2**, (1998), 235–253.
- [3] S. IZUMINO, M. TOMINAGA, *Estimations in Hölder's type inequalities*, Math. Inequal. & Appl., **4**, 2 (2001), 163–187.
- [4] D. MITRINOVIĆ, J. E. PEČARIĆ AND A. M. FINK, *Classical and New Inequalities in Analysis*, Kluwer Academic Publishers, Boston, London 1993.
- [5] N. OZEKI, *On the estimation of the inequalities by the maximum, or minimum values* (in Japanese), J. College Arts Sci, Chiba Univ., **5**, (1968), 199–203.
- [6] E. ZEIDLER, *Applied Functional Analysis; Main Principles and Their Applications*, Applied Mathematical Sciences 109, Springer-Verlag, New York, 1995.