Mathematical
Inequalities
& Applications
Volume 8, Number 3 (2005), 389–402

ON APPROXIMATE t-CONVEXITY

ATTILA HÁZY

Abstract. A real valued function f defined on an open convex set D is called (ε,δ,p,t)-convex if it satisfies

$$f(tx+(1-t)y) \leqslant tf(x)+(1-t)f(y)+\delta+\varepsilon|x-y|^p \qquad \text{for} \quad x,y\in D.$$

The main result of the paper states that if f is locally bounded from above at a point of D and (ε,δ,p,t)-convex (where $0\leqslant p<1$ and $t\leqslant 1/2$) then it satisfies the convexity-type inequality

$$f\Big(\lambda x+(1-\lambda)y\Big) \leqslant \lambda f(x)+(1-\lambda)f(y)+\delta/t+\varepsilon\varphi(\lambda)|x-y|^p \qquad \text{for} \quad x,y\in D,\ \lambda\in[0,1],$$

where $\varphi:[0,1]\to\mathbb{R}$ is a continuous function satisfying

$$\varphi(\lambda)=\max\left\{\frac{1}{t^p-t};\frac{1}{(1/2-t/2)^p-(1-t)^{1-p}(1/2-t)^p}\right\}(\lambda(1-\lambda))^p.$$

In the case $p=1, t=1/2$ analogous results were obtained in [2].

Mathematics subject classification (2000): 26A51, 26B25.
Key words and phrases: convexity, approximately convexity, (ε,δ,p,t)-convexity.

REFERENCES

[1] F. BERNSTEIN AND G. DOETSCH, *Zur Theorie der konvexen Funktionen*, Math. Annalen **76** (1915), 514–526.
[2] A. HÁZY AND ZS. PÁLES, *Approximately midconvex functions*, Bulletin London Math. Soc., **36**, 3 (2004), 339–350.
[3] M. KUCZMA, *An Introduction to the Theory of Functional Equations and Inequalities*, Państwowe Wydawnictwo Naukowe — Uniwersytet Śląski, Warszawa–Kraków–Katowice, 1985.
[4] C. T. NG AND K. NIKODEM, *On approximately convex functions*, Proc. Amer. Math. Soc. **118** 1 (1993), 103–108.
[5] ZS. PÁLES, *Bernstein–Doetsch-type results for general functional inequalities*, Rocznik Nauk.-Dydakt. Prace Mat. **17** (2000), 197–206, Dedicated to Professor Zenon Moszner on his 70th birthday.
[6] S. PICCARD, *Sur des ensembles parfaits*, Mém. Univ. Neuchâtel, **16**, Secrétariat de l'Université, Neuchâtel, 1942
[7] H. STEINHAUS, *Sur les distance des points des ensembles de mesure positif*, Fund. Math. **1** (1920), 99–104.

© ELEMENT, Zagreb
Paper MIA-08-37