

LIAPUNOV–TYPE INEQUALITIES AND NEUMANN BOUNDARY VALUE PROBLEMS AT RESONANCE

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Abstract. This paper is devoted to the study of resonant nonlinear boundary value problems with Neumann boundary condition. First we consider the linear situation doing a careful analysis on the existence of nontrivial solutions. This analysis involves Liapunov-type inequalities with the L_p -norm of the coefficient function for $1 \leq p \leq \infty$. We carry out a complete treatment of the problem for any constant $p \geq 1$. Then, this is combined with Schauder fixed point theorem to obtain new results about the existence and uniqueness of solutions for resonant nonlinear problems.

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