Mathematical
Inequalities
& Applications
Volume 9, Number 2 (2006), 303–324

ON RADIALLY SYMMETRIC SOLUTIONS OF SECOND AND HIGHER ORDER NONLINEAR PARTIAL DIFFERENTIAL EQUATIONS

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Abstract. Explicit radially symmetric entire and non-entire solutions are obtained for the equations of the form

$$\Delta^k u + P(r)f(u) = 0, \quad k \geqslant 1 \tag{1.1}$$

where $P(r)$ is a suitable function, Δ denotes n-dimensional Laplace operator and Δ^k is the k^{th} iterate of Δ. In particular, the cases

$$f(u) = \pm b u^{\frac{n+2k}{n-2k}}$$

where b is a positive constant, are considered. For $n = 2$, infinitely many entire solutions of

$$\Delta u + be^u = 0$$

and non-entire solutions of

$$\Delta u = be^u$$

are derived. Explicit solutions of some nonlinear Dirichlet and Neumann problems and some singular nonlinear ordinary differential equations are also determined. These results are consequence of a differential inequality or appropriately chosen form of the solution.

Mathematics subject classification (2000): 35J30, 35J60, 31B30.
Key words and phrases: Laplace operator, explicit radially symmetric solutions, entire and non-entire solutions, singular ordinary nonlinear differential equations.

REFERENCES

[1] C. BANDLE, *Mean value theorems for functions satisfying the inequality* $\Delta u + ke^u \geqslant 0$, Arch. Rational Mech. Anal., **51**, (1973), 70–84.
[2] L. BIEBERBACH, $\Delta u = e^u$ *und die automorphen funcktionen*, Mathematische Annalen, **77**, (1916), 173–212. See also Gott. Nach. 1912, 599–602.
[3] B. GIDAS, W-M. NI AND L. NIRENBERG, *Symmetry and related properties via the maximum principle*, Comm. Math. Phys., **68**, (1979), 209–243.
[4] S. GOYAL, V. GOYAL, *A problem of prescribing Gaussian Curvature on* S^2, Proc. of Amer-Math. Soc., **129**, (2001), 3757–3758.
[5] S. GOYAL, V. GOYAL AND P. SCHAEFER, *On Solutions of* $\Delta u \geqslant P(r)f(u)$, J. Inequalities Appl., **7**, (2002), 503–515.
[6] J. B. KELLER, *On solutions of* $\Delta u = f(u)$, Comm. Pure Appl. Math., **10**, (1957), 503–510.
[7] C. LOEWNER, L. NIRENBERG, *Partial differential equations invarient under conformal or projective transformations*, in Contributions to Analysis, Academic Press (1974), 245–272.
[8] Z. Nehari, *Bounds for the solutions of a class of nonlinear partial differential equations*, Proc. Amer. Math. Soc., **14**, (1963), 829–836.
[9] R. OSSERMAN, *On the inequality* $\Delta u \geqslant f(u)$, Pacific J. Math., **7**, (1957), 1641–1647.

[10] P. W. SCHAEFER, V. GOYAL, *On explicit entire and non-entire solutions*, The Proceedings of the International Society for Analysis, Applications and computation conference held in Berlin in August 2001. (To appear.)
[11] R. SCHOEN, *Conformal deformation of a Riemannian metric to constant scalar curvature*, J. Differential Geom., **20**, (1984), 479–495.
[12] N. S. TRUDINGER, *Remarks concerning conformal deformation of Riemannian structures on compact manifolds*, Ann. Scuola Norm. Sup. Pisa CI Sci, **4**, 3 (1968), 265–274.
[13] W. WALTER, *Über ganze Lösungen der Differentialgleichung* $\Delta u = f(u)$, J. Ber. Deutsch Math. Verein, **57**, (1955), 94–102.
[14] W. WALTER, *Zur existenz ganzer Lösungen der Differentialgleichung* $\Delta^k u = e^u$, Arch. Math. IX (1958), 308–312.
[15] WEI-MING NI, *On elliptic equation* $\Delta u + K(x)u^{\frac{n+2}{n-2}} = 0$, *its generalizations and applications in geometry*, Indiana University Mathematics Journal, **31**, 4 (1982), 493–529.
[16] H. WITTICH, *Ganze Lösungen der Differentialgleichung* $\Delta u = e^u$, Math. Z., **49**, (1943/44), 579–582.
[17] H. YAMABE, *On a deformation of Riemmanian structures on compact manifolds*, Osaka Math. J., **12**, (1960), 21–37.