

## ON THE INTERMEDIATE POINT IN CAUCHY'S MEAN-VALUE THEOREM

DOREL I. DUCA AND OVIDIU POP

*Abstract.* If the functions  $f, g : I \rightarrow \mathbb{R}$  are differentiable on the interval  $I \subseteq \mathbb{R}$ , then for each  $x, a \in I$  there exists a real number  $\theta \in ]0, 1[$  such that

$$(f(x) - f(a))g^{(1)}(a + \theta(x - a)) = (g(x) - g(a))f^{(1)}(a + \theta(x - a)).$$

In this paper we study the behaviour of the number  $\theta \in ]0, 1[$ , when  $x$  approaches  $a$ .

*Mathematics subject classification (2000):* 26A24.

*Key words and phrases:* Cauchy's theorem, intermediate point, mean-value theorem.

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